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Math 251
22 November 2011

$$z = 5t + \frac{\cos^2(t^2)}{3}$$

$$z' = \left[\frac{d}{dt}5t\right] + \frac{\left[\frac{d}{dt}\left(\cos(t^2)\right)^2\right](3) - \left[\frac{d}{dt}3\right]\left(\frac{\left(\cos(t^2)\right)^2}{3}\right)}{(3)^2}$$

$$= 5 + \frac{\left[\left(2\cos(t^2)\right)\left[\frac{d}{dt}\cos(t^2)\right]\right](3) - (0)\left(\frac{\left(\cos(t^2)\right)^2}{3}\right)}{9}$$

$$= 5 + \frac{\left[\left(2\cos(t^2)\right)\left[\left(-\sin(t^2)\right)(2t)\right]\right](3)}{9}$$

$$z' = 5 - \frac{4\cos(t^2)\sin(t^2)}{3}$$

In **Figure 1** is a demonstration of a rather complex use of the techniques covered so far this semester. The first thing shown is splitting the second term up using the quotient rule. Then, since the second term contains multiplication by zero the term is eliminated, but the first term needs the use of the chain rule to find the derivative. The final step shows that the derivative of *z(t)*= $5 - \frac{4\cos(t^2)\sin(t^2)}{3}$

Note: In **Figure 1** (above) it is a time efficient skill to be able to know when to stop working. There is no need to break things down just because they still look complicated. Sometimes derivatives are more complicated looking then the function they are derived from, but we are not using derivatives to simplify. We now use math for application, we can learn a lot from a graph by taking the derivative of its equation, as demonstrated throughout the past several weeks.

Figure 2

$$G = x\sin^{-1}[x\ln(x)]$$

$$G' = \frac{d}{dx}x\sin^{-1}[x\ln(x)]$$

$$= \left(\frac{d}{dx}x\right)\left(\sin^{-1}[x\ln(x)]\right) + (x)\left(\frac{d}{dx}\sin^{-1}[x\ln(x)]\right)$$

$$= \left(\sin^{-1}[x\ln(x)]\right) + (x)\left(\frac{1}{\sqrt{1-[x\ln(x)]^2}}\right)\left(\frac{d}{dx}[x\ln(x)]\right)$$

$$= \left(\sin^{-1}[x\ln(x)]\right) + (x)\left(\frac{1}{\sqrt{1-[x\ln(x)]^2}}\right)\left(\frac{d}{dx}x\right)\left(\ln(x)\right) + \left(\frac{d}{dx}\ln(x)\right)(x)$$

$$= \left(\sin^{-1}[x\ln(x)]\right) + \left(\frac{x}{1}\right)\left(\frac{1}{\sqrt{1-[x\ln(x)]^2}}\right)\left(\frac{\ln(x)}{1}\right) + \left(\frac{1}{x}\right)\left(\frac{x}{1}\right)$$

$$= \left(\sin^{-1}[x\ln(x)]\right) + \left(\frac{x}{1}\right)\left(\frac{1}{\sqrt{1-[x\ln(x)]^2}}\right)\left(\frac{\ln(x)}{1}\right) + \left(\frac{1}{1}\right)$$

$$= \left(\sin^{-1}[x\ln(x)]\right) + \left(\frac{x(\ln(x)+1)}{\sqrt{1-[x\ln(x)]^2}}\right)$$

In **Figure 2** there are a couple of easy-to-miss steps. The thing to notice is that it is only the use of the product rule along with the chain rule if you know the trigonometric inverse derivatives. This work also demonstrates the use of the natural logarithm derivatives, showing the importance of having these identities and/or formulas memorized. The final step shows that the derivative of G(x)=

$$\left(\sin^{-1}[x\ln(x)]\right) + \left(\frac{x(\ln(x)+1)}{\sqrt{1-[x\ln(x)]^2}}\right)$$

Note: It is important here (**Figure 2**) to realize that many expressions can be written over one therefor multiplying together in the numerator.

Figure 3

$$y = \tan^{-1}(\tan(x))$$
$$y' = \frac{d}{dx}\tan^{-1}(\tan(x))$$
$$= \frac{1}{[\tan(x)]^2+1}\left[\frac{d}{dx}\tan(x)\right]$$
$$= \frac{1}{[\sec(x)]^2}\left(\frac{1}{[\cos(x)]^2}\right)$$
$$= \frac{[\cos(x)]^2}{1}\left(\frac{1}{[\cos(x)]^2}\right)$$
$$= \frac{[\cos(x)]^2}{[\cos(x)]^2}$$
$$= 1$$

Figure 3 shows the proof that the inverse tangent of the tangent of any value is that value. What the derivative being equal to one (as seen in **Figure 3**) tells us about the function is that the slope of the entire function is continuously equal to one. In this case the derivate 1 is proving that the whole equation is equal to the equation *y*=*x*

Figure 4

$$\frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2} \qquad \left[\tan^{-1}(x)\right] \Leftrightarrow \left[\tan(y) = x\right]$$
$$\frac{d}{dx}\tan(x) = \sec^2(x)$$

In **Figure 4** is a list of some things that are key to remember {from previous lectures and classes (i.e., Trigonometry)} for the next process being demonstrated in **Figure 5.**

Figure 5

$$\frac{d}{dx}\tan^{-1}(x) =$$
$$\left(\frac{d}{dx}x\right) = \left(\frac{d}{dx}\tan(y)\right)$$
$$1 = \left[\sec^2(y)\right]\left[\frac{d}{dx}y\right]$$
$$\frac{1}{\sec^2(y)} = \frac{dy}{dx}$$
$$\frac{1}{\tan^2(y)+1} = \frac{dy}{dx}$$
$$\frac{1}{x^2+1} = \frac{dy}{dx}\tan^{-1}(x)$$

In this demonstration, shown to the right in **Figure 5,** it will be necessary to remember key identities from trigonometry (particularly the Pythagorean identity) along with some of the formulas that have been demonstrated previously (i.e., the derivative of the inverse of tangent). All of these have been listed clearly in **Figure 4.** Also included in this proof is the use of implicit differentiation, which is the most difficult concept presented to date.